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The Gluon Distribution as a Function of F_2 and $dF_2/d\ln Q^2$ at small x. The Next-to-Leading Analysis

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Abstract

We present a set of formulae to extract the gluon distribution function from the deep inelastic structure function F_2 and its derivative $dF_2/d\ln Q^2$ at small x in the leading and next-to-leading order of perturbation theory. The detailed analysis is given for new HERA data. The values of the gluon distribution are found in the range $10^{-4} \leq x \leq 10^{-2}$ at $Q^2 = 20$ GeV^2

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The knowledge of the DIS structure functions at small values of the Bjorken scaling variable x is interesting for understanding the inner structure of hadrons. Of great relevance is the determination of the gluon density at low x, where gluons are expected to be dominant, because it could be a test of perturbative QCD or a probe of new effects, and also because it is the basic ingredient in many other calculations of different high energy hadronic processes.

Recently two experiments working in the electron-proton collider HERA at DESY (H1 and ZEUS) have published new data on the structure function F_2 [1], [2]. Up to now all the analysis performed of these data [3] - [5] found that the gluon distribution rises steeply towards low x (in the moderate Q^2 range of the measurements). This behaviour has been recently connected within the DGLAP evolution equations [6] with the less singular are found at lower values of Q^2 by NMC and E665 experiments.

We introduce the standard parameterizations of singlet quark $s(x,Q^2)$ and gluon $g(x,Q^2)$ parton distribution functions (PDF)[3] (see, for example, [5])

$$\begin{aligned} s(x,Q^2) &= A_s x^{-\delta}(1-x)^{\nu_s}(1+\epsilon_s\sqrt{x}+\gamma_s x) \equiv x^{-\delta}\tilde{s}(x,Q^2) \\ g(x,Q^2) &= A_g x^{-\delta}(1-x)^{\nu_g}(1+\epsilon_g\sqrt{x}+\gamma_g x) \equiv x^{-\delta}\tilde{g}(x,Q^2), \end{aligned} \tag{1}$$

with Q^2 dependent parameters in the r.h.s..

Note that the behaviour of Eq. (1) with a Q^2-independent value for δ ($\delta_q = \delta_g$) obeys the DGLAP equation when $x^{-\delta} \gg 1$ (see, for example, [8] - [10]). If $\delta(Q_0^2) = 0$ in some point $Q_0^2 \sim 1GeV^2$ (see [11], [12], [6]) , then the behaviour $p(x,Q^2) \sim Const$ ($p = (s,g)$) is not compatible with DGLAP equation and a more singular behaviour is generated. If we restrict the analysis to a Regge-like form of structure functions, one obtains (see [6])

$$p(x,Q^2) \sim x^{-\delta_p(Q^2)}$$

with next-to-leading order (NLO) $\delta_q(Q^2) \neq \delta_g(Q^2)$ intercept trajectories.

Without any restriction the double-logarithmical behaviour, i.e.

$$p(x,Q^2) \sim \exp\left(\frac{1}{2}\sqrt{\delta_p(Q^2) ln\frac{1}{x}}\right) \tag{2}$$

is generated. At NLO and for $f = 3$ active quarks, $\delta_p(Q^2)$ have the form (see [6]):

$$\delta_g(Q^2) = \frac{4}{3}t - \frac{1180}{81}l, \quad \delta_q(Q^2) = \delta_g(Q^2) - 20l$$

while for $f = 4$ one has:

$$\delta_g(Q^2) = \frac{36}{25}t - \frac{91096}{5625}l, \quad \delta_q(Q^2) = \delta_g(Q^2) - 20l$$

where $t = ln(\alpha(Q_0^2)/\alpha(Q^2))$ and $l = \alpha(Q_0^2) - \alpha(Q^2)$.

Because we would like to get formulae to extract the gluon distribution from experimental data without theoretical restrictions, we will consider both, the Regge-like behaviour of Eq. (1) if $x^{-\delta} \gg 1$, and the non-Regge-like behaviour of Eq. (2) if $\delta(Q_0^2) = 0$.

[3] We use PDF multiplied by x ($s = xS$ and $g = xG$) and neglect the nonsinglet quark distribution at small x

In this work we present a simple relation between the gluon and F_2 which permits the extraction of the gluon directly from data with a NLO precision in perturbative QCD. This kind of formulae (see also the NLO result of ref. [16] which depends on an unknown phenomenological function) are useful as an alternative to the most complex analysis involved in global QCD fits. At low x the coupled integro-differential equations can be converted in more simple linear relations between parton densities and structure functions[4]. The method to arrive to the solution presented below is based in the replacement of the Mellin convolution by ordinary products developed in ref. [15] that was already applied in the derivation of the leading order (LO) formula in ref. [17] and the NLO result at $\delta = 1/2$ in ref. [18].

1. Assuming the *Regge-like behaviour* for the gluon distribution and $F_2(x,Q^2)$:

$$g(x,Q^2) = x^{-\delta}\tilde{g}(x,Q^2), \quad F_2(x,Q^2) = x^{-\delta}\tilde{f}(x,Q^2),$$

we obtain the following equation for the Q^2 derivative of the SF F_2[5]:

$$\frac{dF_2(x,Q^2)}{dlnQ^2} = -\frac{1}{2}x^{-\delta}\sum_{p=s,g}\Big(r^{1+\delta}_{sp}(\alpha)\ \tilde{p}(0,Q^2) + r^{\delta}_{sp}(\alpha)\ x\tilde{p}'(0,Q^2) + O(x^2)\Big), \tag{3}$$

where $r^{\eta}_{sp}(\alpha)$ are the combinations of the anomalous dimensions (AD) of Wilson operators $\gamma^{\eta}_{sp} = \alpha\gamma^{(0),\eta}_{sp} + \alpha^2\gamma^{(1),\eta}_{sp} + O(\alpha^3)$ and Wilson coefficients[6] $\alpha B^{p,\eta}_2 + O(\alpha^2)$ of the η "moment" (i.e., the corresponding variables expanded from integer values of argument to non-integer ones):

$$\begin{aligned} r^{\eta}_{ss}(\alpha) &= \alpha\gamma^{(0),\eta}_{ss} + \alpha^2\Big(\gamma^{(1),\eta}_{ss} + B^{g,\eta}_2\gamma^{(0),\eta}_{gs} + 2\beta_0 B^{s,\eta}_2\Big), \\ r^{\eta}_{sg}(\alpha) &= \frac{e}{f}\Big[\alpha\gamma^{(0),\eta}_{sg} + \alpha^2\Big(\gamma^{(1),\eta}_{sg} + B^{g,\eta}_2(2\beta_0 + \gamma^{(0),\eta}_{gg} - \gamma^{(0),\eta}_{ss})\Big)\Big] \end{aligned} \tag{4}$$

and

$$\tilde{p}'(0,Q^2) \equiv \frac{d}{dx}\tilde{p}(x,Q^2) \text{ at } x=0$$

where $e = \sum_i^f e_i^2$ is the sum of squares of quark charges.

For the gluon part from r.h.s of Eq.(3) with accuracy of $O(x^2)$, we have the form:

$$r^{1+\delta}_{sg}\tilde{g}(x/\xi_{sg},Q^2) \ \text{ with } \ \xi_{sg} = r^{1+\delta}_{sg}/r^{\delta}_{sg} \tag{5}$$

In the quark part a similar simple form is absent because the corresponding LO anomalous dimensions, $\gamma^{(0),1+\delta}_{ss}$ and $\gamma^{(0),\delta}_{ss}$, have opposite signs. However, within accuracy $O(x^2)$, it may be represented as a sum of two terms like Eq.(5), with a shift of some coefficients and arguments. Choosing the shifts as 1 and A we have the following representation for the quark part:

$$c_1\ \tilde{s}(x,Q^2)\ +\ c_2\ \tilde{s}(Ax,Q^2)\ +\ O(Ax^2),$$

[4] There are analogous formulae connecting F_L with F_2 and its derivative and also for the extraction of gluons from F_L (see the at LO refs.[13] and [14] and at NLO refs. [7] and [15], respectively)

[5] Hereafter contrary to the standard case we use $\alpha(Q^2) = \alpha_s(Q^2)/4\pi$.

[6] Because we consider here $F_2(x,Q^2)$ but not the singlet quark distribution

where

$$c_2 \;=\; \frac{r^{\delta}_{ss} - r^{1+\delta}_{ss}}{A-1} \quad \text{and} \quad c_1 \;=\; \frac{Ar^{1+\delta}_{ss} - r^{\delta}_{ss}}{A-1} \tag{6}$$

We thus find the following expression from Eqs. (3)-(6)

$$\frac{dF_2(x,Q^2)}{dlnQ^2} = -\frac{1}{2}\Bigl[r^{1+\delta}_{sg}(\xi_{sg})^{-\delta} g(x/\xi_{sg},Q^2) + c_1 F_2(x,Q^2) + c_2 F_2(Ax,Q^2)\Bigr] \\ +\; O(x^{2-\delta}, \alpha A x^{2-\delta}), \tag{7}$$

because $c_i \sim O(\alpha)$.

From Eq. (7) with the accuracy of $O(x^{2-\delta}, \alpha A x^{2-\delta})$, we obtain for gluon PDF:

$$g(x,Q^2) = -\frac{(\xi_{sg})^{\delta}}{r^{1+\delta}_{sg}}\Bigl[2\cdot\frac{dF_2(x\xi_{sg},Q^2)}{dlnQ^2} + c_1 F_2(x\xi_{sg},Q^2) + c_2 F_2(Ax\xi_{sg},Q^2) \\ +\; O(x^{2-\delta}, \alpha A x^{2-\delta})\Bigr] \tag{8}$$

Because the value of A is arbitrary it is convenient to neglect the contribution from $F_2(x\xi_{sg}/A,Q^2)$. Putting formally $A=\infty$[7], one arrives to the final general formula to extract $g(x,Q^2)$ in the NLO approximation:

$$g(x,Q^2) = -\frac{(\xi_{sg})^{\delta}}{r^{1+\delta}_{sg}}\Bigl[2\cdot\frac{dF_2(x\xi_{sg},Q^2)}{dlnQ^2} \;+\; r^{1+\delta}_{ss}\; F_2(x\xi_{sg},Q^2) \;+\; O(x^{2-\delta}, \alpha x^{1-\delta})\Bigr] \tag{9}$$

Restricting the analysis to $O(x^{2-\delta}, \alpha x^{1-\delta})$, one can replace $\xi_{sg} \to \xi = \gamma^{(0),1+\delta}_{sg}/\gamma^{(0),\delta}_{sg}$ into Eq. (9):

$$g(x,Q^2) = -\frac{\xi^{\delta}}{r^{1+\delta}_{sg}}\Bigl[2\cdot\frac{dF_2(x\xi,Q^2)}{dlnQ^2} \;+\; r^{1+\delta}_{ss}\; F_2(x\xi,Q^2) \;+\; O(x^{2-\delta}, \alpha x^{1-\delta})\Bigr] \tag{10}$$

This replacement is very useful. The NLO AD $\gamma^{(1),n}_{sp}$ are singular[8] in both points, $n=1$ and $n=0$, and their presence into the arguments of $\tilde{p}(x,Q^2)$ makes the numerical agreement between this approximate formula and the exact calculation worse (we have checked this point using some MRS sets of parton distributions).

Using NLO approximation of $r^{1+\delta}_{sp}$ we easily obtain the final results for $g(x,Q^2)$:

$$g(x,Q^2) = -\frac{2f}{\alpha e}\frac{\xi^{\delta}}{\gamma^{(0),1+\delta}_{sg} + \overline{\gamma}^{(1),1+\delta}_{sg}\alpha}\Bigl[\frac{dF_2(x\xi,Q^2)}{dlnQ^2} \\ +\; \frac{\alpha}{2}\cdot\left\{\begin{array}{ll} \Bigl(\gamma^{(0),1+\delta}_{ss} + \overline{\gamma}^{(1),1+\delta}_{ss}\alpha\Bigr)F_2(x\xi,Q^2) & +\;\; O(x^{2-\delta}, \alpha x^{1-\delta}) \\ \gamma^{(0),1+\delta}_{ss}F_2(x\xi,Q^2) & +\;\; O(\alpha^2, x^{2-\delta}, \alpha x^{1-\delta}) \end{array}\right.\Bigr] \tag{11}$$

$$g(x,Q^2) = -\frac{2f}{\alpha e}\frac{1}{\gamma^{(0),1+\delta}_{sg} + \overline{\gamma}^{(1),1+\delta}_{sg}\alpha}\Bigl[\frac{dF_2(x,Q^2)}{dlnQ^2} \\ +\; \frac{\alpha}{2}\cdot\left\{\begin{array}{ll} \Bigl(\gamma^{(0),1+\delta}_{ss} + \overline{\gamma}^{(1),1+\delta}_{ss}\alpha\Bigr)F_2(x,Q^2) & +\;\; O(x^{1-\delta}) \\ \gamma^{(0),1+\delta}_{ss}F_2(x,Q^2) & +\;\; O(\alpha^2, x^{1-\delta}) \end{array}\right.\Bigr] \tag{12}$$

[7]Really $A \sim (x\xi_{sg})^{-1}$.

[8]In the case of replacement Mellin convolution by ordinary product these singularities transform to logarithmically increasing terms (see [9] and [15])

where

$$\overline{\gamma}_{sg}^{(1),\eta} = \gamma_{sg}^{(1),\eta} + B_2^{g,\eta}(2\beta_0 + \gamma_{gg}^{(0),\eta} - \gamma_{ss}^{(0),\eta})\Big)$$

$$\overline{\gamma}_{ss}^{(1),\eta} = \gamma_{ss}^{(1),\eta} + B_2^{g,\eta}\gamma_{gs}^{(0),\eta} + 2\beta_0 B_2^{s,\eta}$$

Any equation from above formulae (10) may be used, because there is a strong cancelation between the shifts in the arguments of the function F_2 and its derivative and the shifts in the coefficients in front of them.

For concrete values of $\delta = 0.5$ (see also [18]) and $\delta = 0.3$ we obtain (for f=4 and $\overline{MS}$ scheme):

if $\delta = 0.5$

$$g(x,Q^2) = \frac{105}{4e\sqrt{23\times 77}}\frac{1}{\alpha}\frac{1}{(1+26.93\alpha)}\Bigl[\frac{dF_2(\frac{23}{77}x,Q^2)}{dlnQ^2} + \frac{16}{3}\alpha\Bigl(\frac{107}{60} - 2ln2\Bigr)\left\{\begin{array}{ll} \Bigl\{1+37.76\alpha\Bigr\}F_2(\frac{23}{77}x,Q^2) & + \; O(x^{2-\delta},\alpha x^{1-\delta}) \\ F_2(\frac{23}{77}x,Q^2) & + \; O(\alpha^2,x^{2-\delta},\alpha x^{1-\delta}) \end{array}\right.\Bigr] \quad (13)$$

$$g(x,Q^2) = \frac{105}{92e}\frac{1}{\alpha}\frac{1}{(1+26.93\alpha)}\Bigl[\frac{dF_2(x,Q^2)}{dlnQ^2} + \frac{16}{3}\alpha\Bigl(\frac{107}{60} - 2ln2\Bigr)\left\{\begin{array}{ll} \Bigl\{1+37.76\alpha\Bigr\}F_2(x,Q^2) & + \; O(x^{1-\delta}) \\ F_2(x,Q^2) & + \; O(\alpha^2,x^{1-\delta}) \end{array}\right.\Bigr] \quad (14)$$

if $\delta = 0.3$

$$g(x,Q^2) = \frac{0.60}{\alpha e}\frac{1}{(1+52.52\alpha)}\Bigl[\frac{dF_2(x/5.27,Q^2)}{dlnQ^2} + 1.89\alpha\left\{\begin{array}{ll} \Bigl\{1-50.67\alpha\Bigr\}F_2(x/5.27,Q^2) & + \; O(x^{2-\delta},\alpha x^{1-\delta}) \\ F_2(x/5.27,Q^2) & + \; O(\alpha^2,x^{2-\delta},\alpha x^{1-\delta}) \end{array}\right.\Bigr] \quad (15)$$

$$g(x,Q^2) = \frac{0.98}{\alpha e}\frac{1}{(1+52.52\alpha)}\Bigl[\frac{dF_2(x,Q^2)}{dlnQ^2} + 1.89\alpha\left\{\begin{array}{ll} \Bigl\{1-50.67\alpha\Bigr\}F_2(x,Q^2) & + \; O(x^{1-\delta}) \\ F_2(x,Q^2) & + \; O(\alpha^2,x^{1-\delta}) \end{array}\right.\Bigr] \quad (16)$$

2. Assuming the *non-Regge-like behaviour* for the gluon distribution and $F_2(x,Q^2)$:

$$g(x,Q^2) = \frac{\exp\,(\frac{1}{2}\sqrt{\delta_g(Q^2)ln\frac{1}{x}})}{(2\pi\delta_g(Q^2)ln\frac{1}{x})^{1/4}}\tilde{g}(x,Q^2), \quad F_2(x,Q^2) = \frac{\exp\,(\frac{1}{2}\sqrt{\delta_s(Q^2)ln\frac{1}{x}})}{(2\pi\delta_s(Q^2)ln\frac{1}{x})^{1/4}}\tilde{f}(x,Q^2),$$

we obtain the following equation for the Q^2 derivative of the SF F_2[9]:

$$\frac{dF_2(x,Q^2)}{dlnQ^2} = -\frac{1}{2}\sum_{p=s,g}\frac{\exp\left(\frac{1}{2}\sqrt{\delta_p(Q^2)ln\frac{1}{x}}\right)}{\left(2\pi\delta_p(Q^2)ln\frac{1}{x}\right)^{1/4}}\Bigl(\tilde{r}^1_{sp}(\alpha)\ \tilde{p}(0,Q^2) + O(x^1)\Bigr), \qquad (17)$$

where $\tilde{r}^1_{sp}(\alpha)$ can be obtained from corresponding functions $r^{1+\delta}_{sp}(\alpha)$ replacing the singular term $1/\delta$ at $\delta \to 0$ by another term $1/\tilde{\delta}$:

$$\frac{1}{\delta} \stackrel{\delta\to 0}{\to} \frac{1}{\tilde{\delta}} \ = \ \sqrt{\frac{ln(1/x)}{\delta_p(Q^2)}} - \frac{1}{4\delta_p(Q^2)}\left[1 + \sum_{m=1}^{\infty}\frac{1\times 3\times ...\times(2m-1)}{\left(4\sqrt{\delta_p(Q^2)\ln(1/x)}\right)^m}\right] \qquad (18)$$

The singular term appears only in the NLO part of the AD $\gamma^{(1),1+\delta}_{sp}$ in Eq. (4). The replacement (18) corresponds to the following transformation:

$$\gamma^{(1),1+\delta}_{sp} \equiv \hat{\gamma}^{(1),1}_{sp}\frac{1}{\delta} + \check{\gamma}^{(1),1+\delta}_{sp} \quad \stackrel{\delta\to 0}{\to} \quad \tilde{\gamma}^{(1),1}_{sp} = \hat{\gamma}^{(1),1}_{sp}\frac{1}{\tilde{\delta}} + \check{\gamma}^{(1),1}_{sp}, \qquad (19)$$

where $\hat{\gamma}^{(1),1}_{sp}$ and $\check{\gamma}^{(1),1+\delta}_{sp}$ are the coefficients corresponding to singular and regular parts of $\gamma^{(1),1+\delta}_{sp}$, respectively.

We restrict here our calculations to $O(x)$ because at $O(x^2)$ one obtains an additional factor:

$$\frac{\exp\left(\frac{1}{2}\sqrt{\delta_g(Q^2)ln\frac{1}{x\xi_{sg}}}\right)}{\left(2\pi\delta_g(Q^2)ln\frac{1}{x\xi_{sg}}\right)^{1/4}}\frac{\left(2\pi\delta_g(Q^2)ln\frac{1}{x}\right)^{1/4}}{\exp\left(\frac{1}{2}\sqrt{\delta_g(Q^2)ln\frac{1}{x}}\right)}$$

in front of the function F_2 and its derivative. This factor complicates very much the final formulae.

Repeating the analysis of the previous section step by step using the replacement (19), we get (for f=4):

$$g(x,Q^2) = \frac{3}{4e}\frac{1}{\alpha}\frac{1}{(1+26\alpha[1/\tilde{\delta}-\frac{41}{13}])}\Biggl[\frac{dF_2(x,Q^2)}{dlnQ^2} + \left\{\begin{array}{lll}\alpha^2\Bigl\{203-61/\tilde{\delta}\Bigr\}F_2(x,Q^2) & + & O(x^1)\\ 0 & + & O(\alpha^2,x^1)\end{array}\right.\Biggr], \qquad (20)$$

because $\gamma^{(0)1}_{ss} = 0$.

3. In the case $x^{-\delta} \gg Const$ our formula (12) at accuracy $O(x^{1-\delta})$ coincides with the corresponding Ellis-Kunszt-Levin result from ref. [10]. By other part, for $\delta = 1$ one arrives to

$$g(x,Q^2) = \frac{3}{4e\alpha}\frac{1}{(1+\frac{619}{108}\alpha)}\Biggl[\frac{dF_2(x/2,Q^2)}{dlnQ^2} + \frac{32}{9}\alpha\left\{\begin{array}{lll}\Bigl\{1+\Bigl[\frac{26231}{1728}-\frac{1181}{576}f\Bigr]\alpha\Bigr\}F_2(x/2,Q^2) & + & O(x^{2-\delta},\alpha x^{1-\delta})\\ F_2(x/2,Q^2) & + & O(\alpha^2,x^{2-\delta},\alpha x^{1-\delta})\end{array}\right.\Biggr], \qquad (21)$$

[9]Using a lower approximation $O(x)$ is not very exact, because in this case F_2 and the gluon distribution can contain an additional factor in the form of a serie $1+\sum_k(1/\delta_p/ln(1/x))^k$, which is determined by boundary conditions (see discussion in Ref.[12]). We will not consider the appearance of this factor in our analysis

that coincides with Prytz results [16] in the LO approximation, when we neglect the contributions $\sim F_2(x,Q^2)$. Both formulae, ((21) and one from [16]), are similar to ours in the NLO case, too. Certainly, the value $\delta = 1$ lies outside the more standard predicted range $0 \leq \delta \leq 1/2$, however in the case of large δ ($\delta > 0.25$) the final formula (10) depends very slowly from the concrete value of δ. This is due to the strong cancelation between the shifts in the arguments and in the coefficients in front of the functions.

Equations (12) and (20) can be combined in a more general formula valid for any value of δ:

$$g(x,Q^2) = -\frac{2f}{\alpha e}\frac{1}{\gamma_{sg}^{(0),1+\delta} + \tilde{\gamma}_{sg}^{(1),1+\delta}\alpha}\Biggl[\frac{dF_2(x,Q^2)}{dlnQ^2} + \frac{\alpha}{2}\cdot\begin{cases} \left(\gamma_{ss}^{(0),1+\delta} + \tilde{\gamma}_{ss}^{(1),1+\delta}\alpha\right)F_2(x,Q^2) & + \; O(x^{1-\delta}) \\ \gamma_{ss}^{(0),1+\delta}F_2(x,Q^2) & + \; O(\alpha^2,x^{1-\delta}) \end{cases}\Biggr] \tag{22}$$

where $\tilde{\gamma}_{sg}^{(1),1+\delta}$ coincides with $\overline{\gamma}_{sg}^{(1),1+\delta}$ with the replacement:

$$\frac{1}{\delta} \to \int_x^1 \frac{dy}{y}\,\frac{g(y,Q^2)}{g(x,Q^2)} \tag{23}$$

In the cases $x^{-\delta} \gg Const$ and $\delta \to 0$ the r.h.s. of (23) leads to $1/\delta$ and $1/\tilde{\delta}$, respectively.

4. In Fig. 1 it is shown the accuracy of Eqs. (13), (15) (both $O(\alpha^2, x^{2-\delta}, \alpha x^{1-\delta})$) and (20) ($O(\alpha^2, x^1)$) in the reconstruction of various gluon distributions from MRS sets at Q^2=20 GeV^2. We have chosen for this test MRS(D_0) (δ=0), MRS(D_-) (δ=0.5) and MRS(G) (δ=0.3) as three representative densities (see ref. [5] and references therein). It can be observed in Fig. 1a that using the formula with $\delta = 0.5$ one gets the best agreement with the input parameterization (less than 1 %) in the case of MRS(D_-) set; for MRS(G) the reconstruction is still good (less than 10 %), but for MRS(D_0) the deviation reach a 30 % at low x.

In Fig. 1b the degree of accuracy of the reconstruction formula with $\delta = 0.3$ can be observed. Here one should expect the set MRS (G) to give also a very good ($\sim 1\%$ level) agreement, however this is not the case because set (G) distinguishes the exponents of the sea-quark part $\delta_s \sim 0$ from the gluon density ($\delta = 0.3$). Thus, Eq. (12) might be slightly modified to treat this case.

Figs. 1c and 1d deal with the case $\delta = 0$. As in Fig. 1a, one can observe a very good accuracy in the reconstruction when Q_0^2 coincides with that of the test parameterization (4 GeV^2 for MRS set). Notice also the lost of accuracy at high x due to the importance of the O(x) terms neglected in Eq. (16)

With the help of Eq. (14) we have extracted the gluon distribution from HERA data, using the slopes $dF_2/dlnQ^2$ determined in ref. [3] and ref. [4]. When H1 data are used the value of F_2 in Eq. (14) was directly taken from the parameterization given by H1 in ref. [1]. With ZEUS data we substitute directly the F_2 values presented in table 1 of their ref. [4]. We have checked that the use of the H1 parameterization for F_2 when dealing with ZEUS data, does not change significantly the $xG(x,Q^2)$ result.

Figures 2a and 2b shows the extracted values of the gluon distribution. It can be observed that the agreement within the errors between the bands, generated from a global fit to data, the parameterization MRS(G), and the extracted points is excellent.

5. In conclusion, a set of new formulae connecting the gluon density with F_2 at low x have been presented. They work fairly well for singular type gluons ($\delta \sim 0.5 - 0.3$) and for the non-singular case ($\delta \sim 0$). We have reproduced previous results of Prytz [16] using $\delta = 1$, and of Ellis-Kunszt-Levin [10] with accuracy $O(x^{1-\delta})$.

We have found that for singular type of gluons the results do not depend practically on the concrete value of the slope δ: there is a cancelation between the changes in the arguments and in coefficients in front of the functions. However, when $\delta \to 0$ the coefficients in front of $dF_2(x,Q^2)/dlnQ^2$ and $F_2(x,Q^2)$ have singularities leading to terms $\sim \sqrt{ln(1/x)}$[10]. Consequently, before to apply these formulae, some fit (maybe quite crude) of experimental data is necessary to verify the type of $F_2(x)$ asymptotic at $x \to 0$.

The formulae were used to generate the gluon distribution that agree with the rise observed by H1 and ZEUS experiments. Further work is in progress in order to obtain similar expressions connecting F_L, F_2 and the Q^2 derivative of F_2.

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References

[1] T. Ahmed et al., H1 Collaboration, Nucl. Phys. B439 (1995) 471.

[2] M. Derrick et al., ZEUS Collaboration, Z. Phys. C65 (1995) 379.

[3] T. Aid et al., H1 Collaboration, Phys. Lett. B354 (1995) 494.

[4] M. Derrick et al., ZEUS Collaboration, Phys. Lett. B345 (1995) 576.

[5] A.D. Martin, W.J. Stirling and R.G. Roberts, *Phys.Lett.* **B354** (1995) 155.

[6] A.V. Kotikov, preprint ENSLAPP-A-519/95 (hep-ph/9504357); preprint hep-ph/9507320.

[7] A.V. Kotikov and G. Parente, work in progress.

[8] F. Martin, Phys. Rev. D19 (1979) 1382; C. Lopez and F.J. Yndurain, Nucl. Phys. B171 (1980) 231; V.I. Vovk, S.J. Maximov and A.V. Kotikov, Teor. Mat. Fiz. 84 (1990) N1, 101.

[9] A.V. Kotikov, Yad. Fiz. 56 (1993) N9, 217.

[10] This happens in the framework of the double-logarithmic asymptotic. The singularities lead to terms $\sim ln(1/x)$ in the case of Regge-like asymptotic (see [6], [9], [15]).

[10] R.K. Ellis, Z. Kunszt and E.M. Levin, Nucl. Phys. B420 (1994) 517.

[11] M. Arneodo et al., NMC Collaboration, preprint CERN-PPE/95-138.

[12] R.D.Ball and S.Forte, *Phys.Lett.* **B336** (1994) 77; **B335** (1994) 77.

[13] A.V. Kotikov, JETP 80 (1995) 979.

[14] A.M. Cooper-Sarkar et al., Z. Phys. C39 (1988) 281.

[15] A.V. Kotikov, Phys. Rev. D49 (1994) 5746; Yad. Fiz. 57 (1994) 1.

[16] K. Prytz, Phys. Lett. B311 (1993) 286; B393 (1994) 393.

[17] A.V. Kotikov, JETP Lett. 59 (1994) 667.

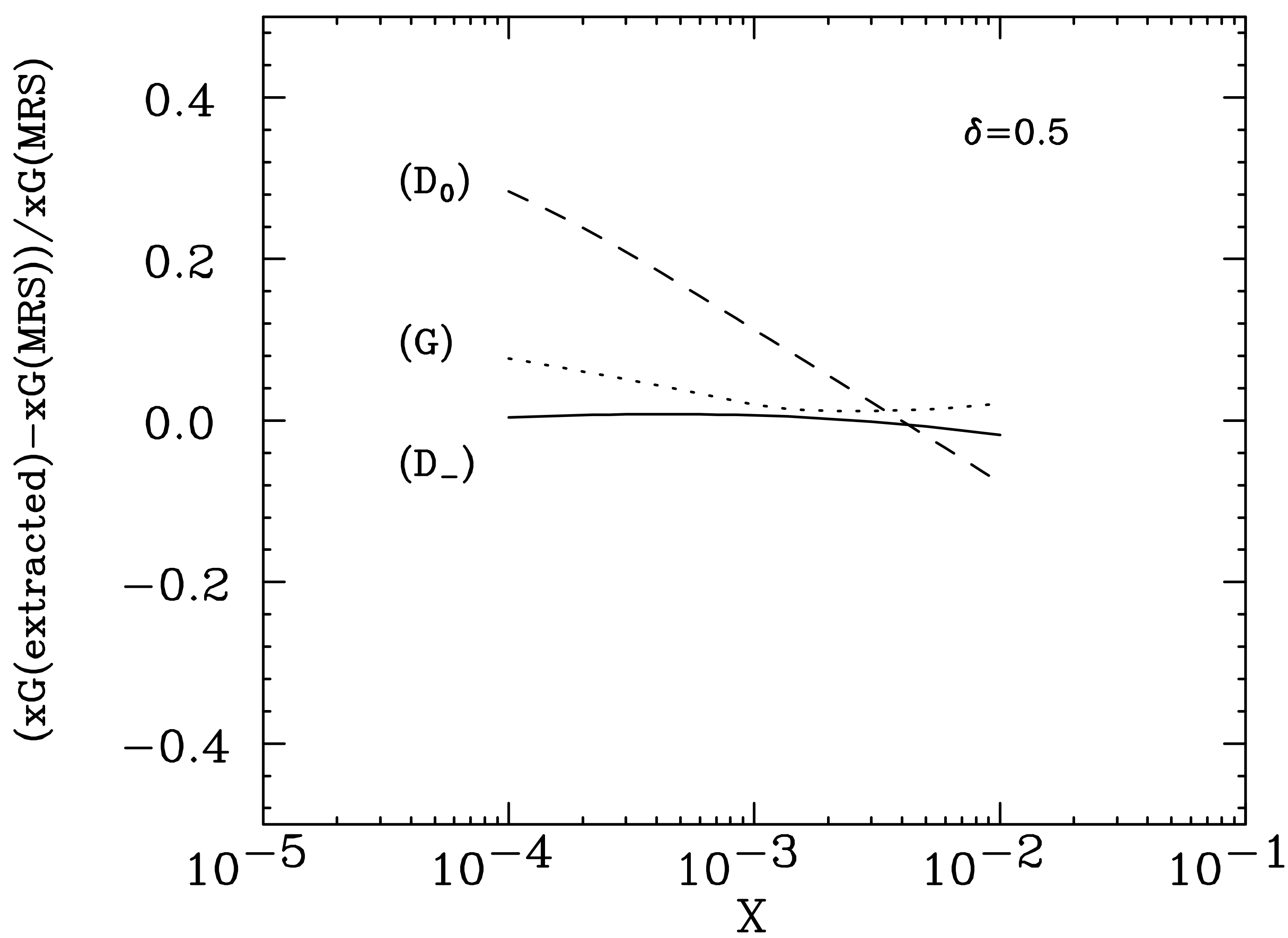
[18] A.V. Kotikov and G. Parente, to appear in the Proceedings of the Int. Europhys. Conf. on High Energy Physics, Brussels, July 27 - Aug. 2, 1995.

Figure captions

Figure 1: Relative difference between the reconstructed gluon distribution using formulae in text and different input parameterizations

Figure 2: The gluon density. The points were extracted from Eq. (14) using H1 (Fig. 2a) and ZEUS (Fig. 2b) data. The dashed curves shows the limits of the error band taken from Fig. 3a of paper [3] and Fig. 4 in ref. [4] which represents the uncertainty from a NLO fit. Solid line is the gluon density from set MRS(G) [5]

Fig. 1a



(xG(extracted)−xG(MRS))/xG(MRS)
0.4
0.2
0.0
−0.2
−0.4
δ=0.5
(D0)
(G)
(D_)
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
X

Fig. 1b

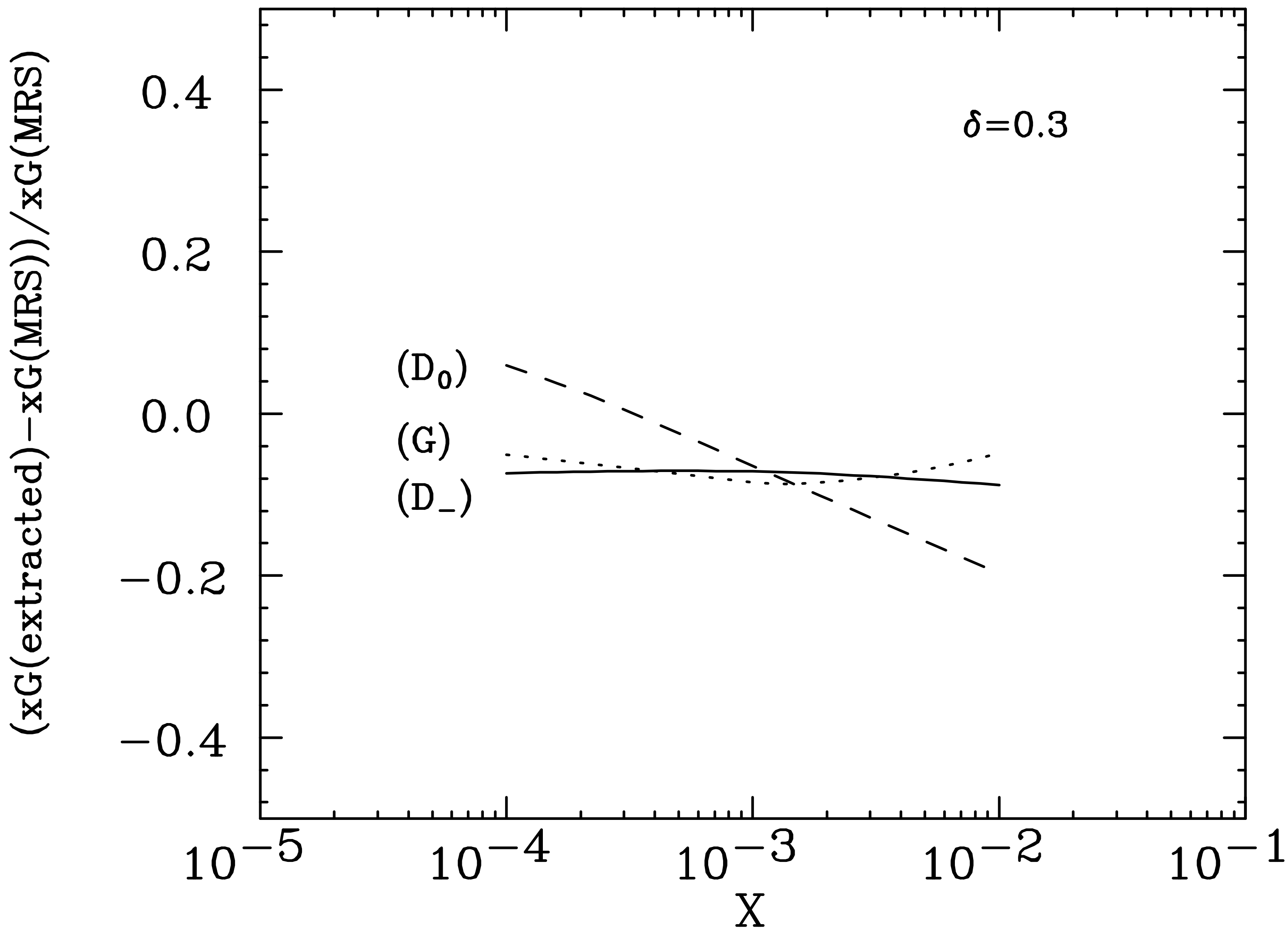
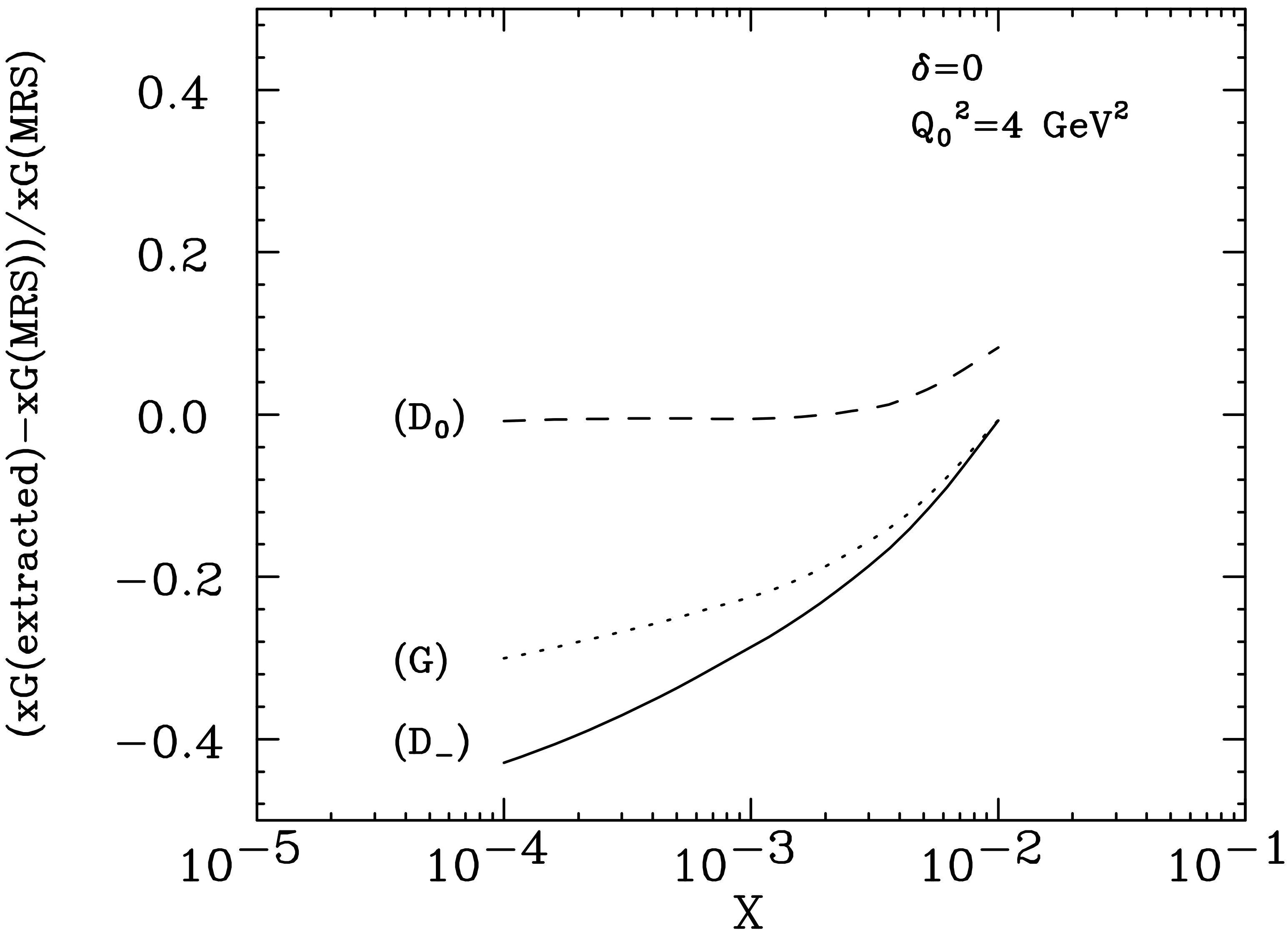

δ=0.3
(D0)
(G)
(D_)
0.4
0.2
0.0
−0.2
−0.4
(xG(extracted)−xG(MRS))/xG(MRS)
10−5
10−4
10−3
10−2
10−1
X

Fig. 1c



(xG(extracted)−xG(MRS))/xG(MRS)
0.4
0.2
0.0
−0.2
−0.4
δ=0
Q_0^2=4 GeV^2
(D_0)
(G)
(D_-)
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
X

Fig. 1d

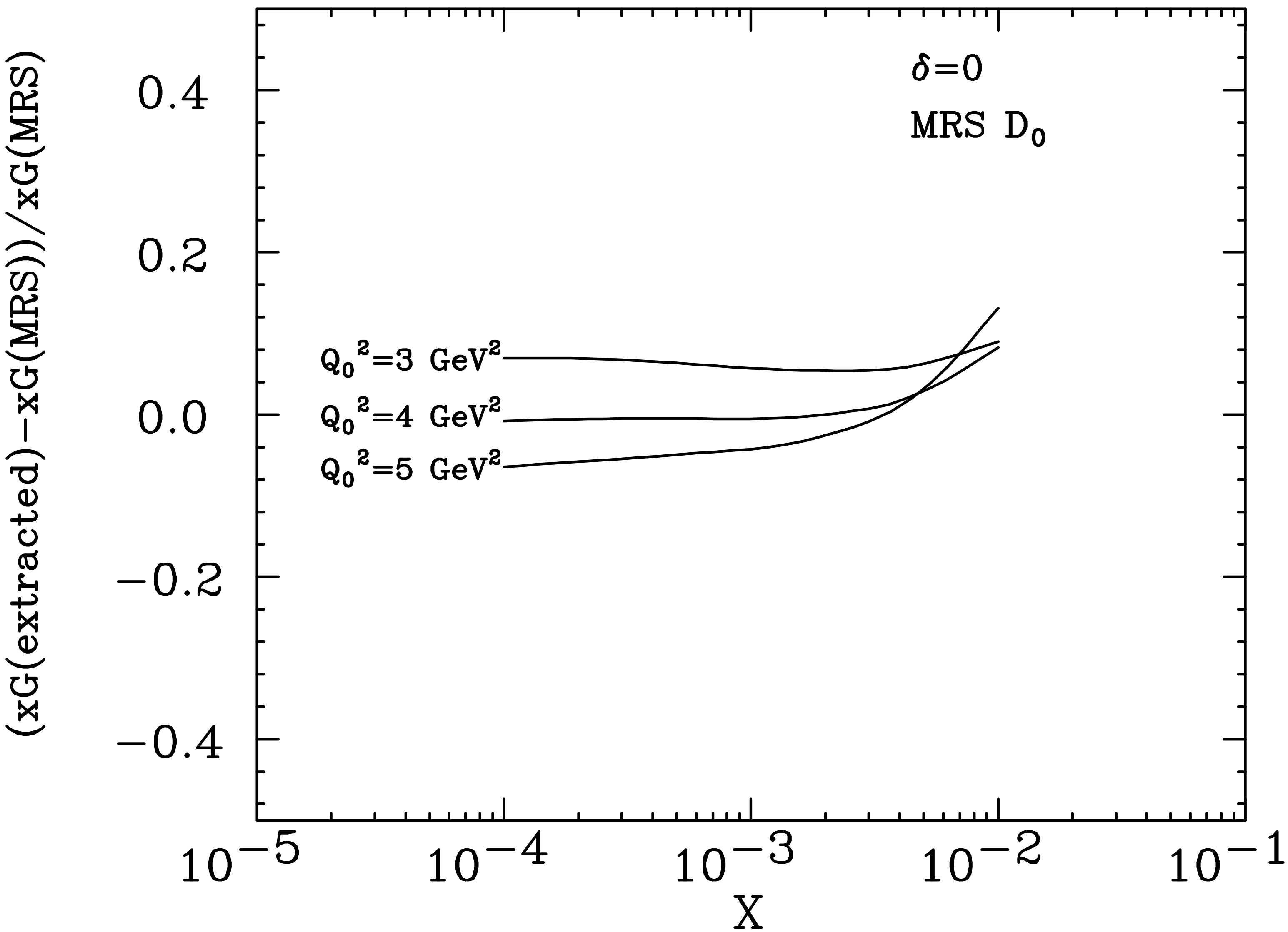

δ=0
MRS D0
Q0²=3 GeV²
Q0²=4 GeV²
Q0²=5 GeV²
0.4
0.2
0.0
−0.2
−0.4
(xG(extracted)−xG(MRS))/xG(MRS)
10−5
10−4
10−3
10−2
10−1
X

Fig. 2a

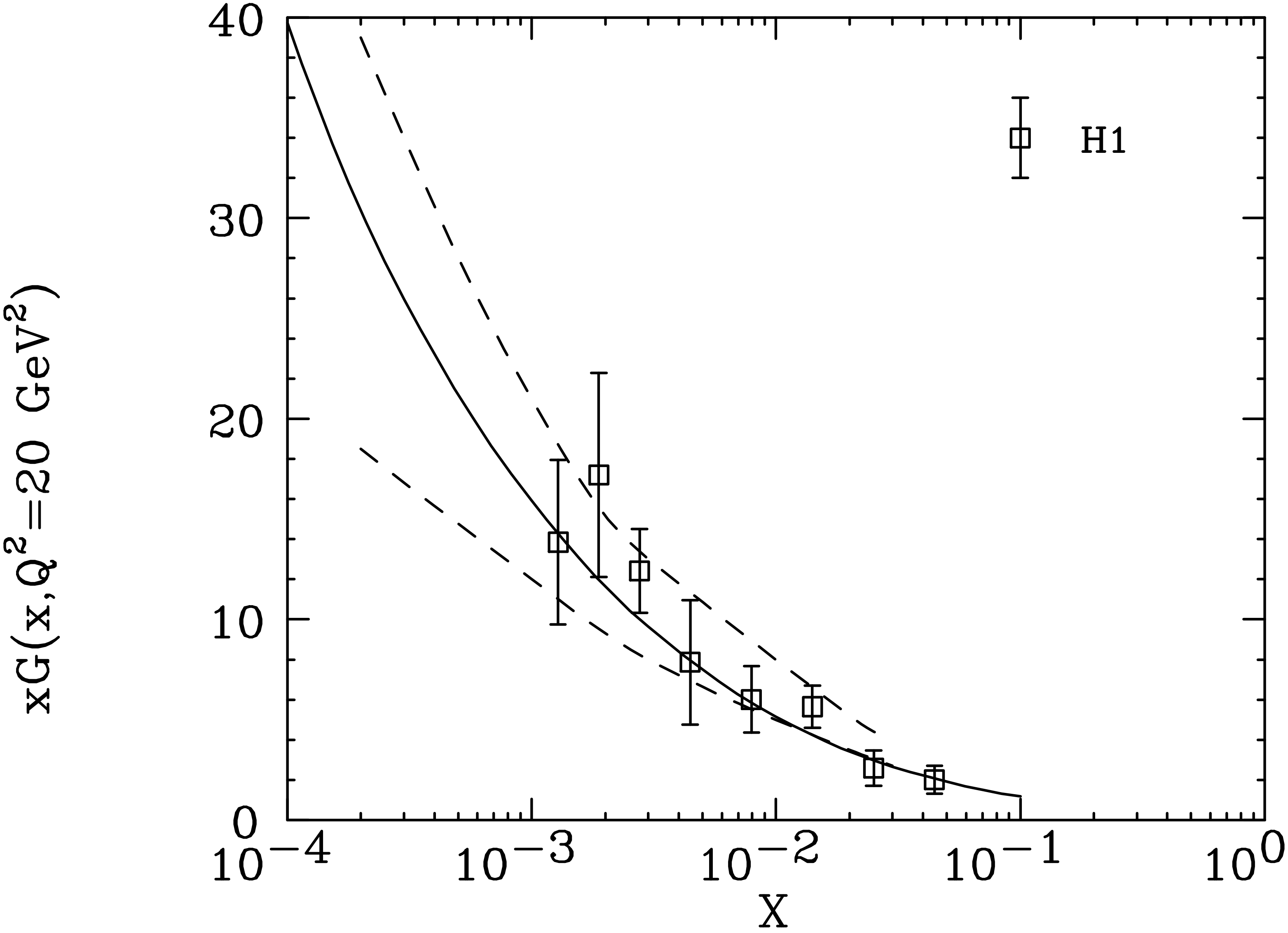


Fig. 2b